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Sequence and series maths

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This reveals that the sequence starts at 3 and increases by 2 each time. Mathematics allows for multiple rules to govern a single sequence. In fact, another possible rule for this sequence is "odd numbers without a 1 in them", which yields {3, 5, 7, 9, 23, 25, ...}. This highlights the importance of specifying "A Rule" instead of "The Rule". To simplify working with rules, mathematicians often use notation such as x_n to represent the term number. For instance, the rule for the sequence {3, 5, 7, 9, ...} can be written as $x_n = 2n+1$. Special sequences like arithmetic and geometric sequences have unique properties. Arithmetic sequences involve adding a constant value each time, while geometric sequences involve multiplying by a constant factor. Triangular numbers form another special sequence, with each term being the sum of previous terms plus one. Examples are provided to illustrate these concepts: * Calculating the 10th term in an arithmetic sequence: $x_{10} = 2n+1 = 2 \times 10+1 = 21$ * Calculating the first 4 terms of a geometric sequence: $\{a_n\} = \{(-1/n)^n\}$ * Calculating the 50th term in a sequence with rule $x_n = 2n+1$ These examples demonstrate how rules can be used to generate and analyze various sequences. By examining the pattern of dots, we can discover the next number in the sequence. Alternatively, use the formula: $x_n = n(n+1)/2$ Example: The fifth triangular number is $x_5 = 5(5+1)/2 = 15$, and the sixth is $x_6 = 6(6+1)/2 = 21$ Square Numbers: 1, 4, 9, 16, 25, ... The next number is found by squaring its position in the pattern. $x_n = n^2$ Cube Numbers: 1, 8, 27, 64, 125, ... The next number is found by cubing its position in the pattern. $x_n = n^3$ Fibonacci Sequence: 0, 1, 1, 2, 3, 5, 8, 13, ... The next number is found by adding the two preceding numbers together: $x_2 = 1+1$, $x_{21} = 8+13$, etc... $x_n = x_{n-1} + x_{n-2}$ This formula is interesting because it relies on the values of the previous two terms. Such recursive formulas are fascinating due to their dependence on earlier terms. The Fibonacci Sequence starts from 0 and continues indefinitely: $n = 0$ 1 2 3 4 5 6 7 8 9 10 11 12 13 ... $x_n = 0$ 1 1 2 3 5 8 13 21 34 55 89 144 233 377 ... For example, term "6" is calculated as: $x_6 = x_{6-1} + x_{6-2} = x_5 + x_4 = 5 + 3 = 8$ Series and Partial Sums: Understanding Sequences Now that you know about sequences, it's time to learn about summing them up. Check out our page on Partial Sums for more information. When we add only part of a sequence, it's called a Partial Sum. However, when we add an infinite sequence, it's called a "Series" (which sounds like another name for sequence, but is actually a sum). Example: {1, 3, 5, 7, ...} Series: $1 + 3 + 5 + 7 + \dots$ Partial Sum of the first three terms: $1 + 3 + 5$ Sequence and series are fundamental concepts in arithmetic. A sequence is an ordered list of objects that can repeat itself, whereas a series is the sum of all elements in a sequence. The key difference between sets and sequences is that individual terms in a sequence can occur repeatedly. The length of a sequence is the number of terms it contains, which can be either finite or infinite. This concept is explored in more detail in Class 11 Maths. With the help of definitions, formulas, and examples, we will discuss the concepts of sequences and series in this article. Check out our study guide for more information: Study Mathematics 1. If a sequence has numbers in a particular order following some rule, then we can define it based on the number of terms, whether finite or infinite. 2. For a given sequence, such as 1, 2, 3, 4,, the corresponding series is represented by $S_N = a_1 + a_2 + a_3 + \dots + a_N$. 3. Finite sequences and infinite sequences are defined based on the number of terms in each type. 4. Common examples of sequences include arithmetic sequences, geometric sequences, harmonic sequences, and Fibonacci numbers. 5. An arithmetic sequence follows a pattern where each term is created by adding or subtracting a definite number to the preceding number. 6. A geometric sequence follows a pattern where each term is obtained by multiplying or dividing a definite number with the preceding number. 7. Harmonic sequences occur when the reciprocals of all elements in the sequence form an arithmetic sequence. 8. Fibonacci numbers are defined as $F_0 = 0$, $F_1 = 1$, and $F_n = F_{n-1} + F_{n-2}$. 9. The general term (nth term) for these sequences can be represented by a formula. 10. Sequences can be differentiated from series based on the order of elements. 11. Finite sequences have a specific number of terms, while infinite sequences extend infinitely. 12. Series do not follow an order as important to sequence definitions. $T_n = ar^{(n-1)}$ where a is the initial term, r is the common ratio, and n is the number of terms. In this case, a = 1, r = 4, and n = 9, so T_9 can be calculated as $48 = 65536$. Sequences are ordered arrangements of numbers, while series are sums of sequence elements. Examples include arithmetic sequences (a, a+d, a+2d, ...), geometric sequences (a, ar, ar², ...), and harmonic sequences. A finite sequence has a last term, whereas an infinite sequence is endless. The common difference in an arithmetic sequence is calculated using the formula: Common difference = Successive term - Preceding term. Arithmetic sequences can be represented as a, a+d, a+2d,, while geometric sequences are represented as a, ar, ar², ... Series can also be arithmetic or geometric. A sequence can be defined by a formula for the nth term (like $a_n^2 - n$) or through an iterative relation (like $a_{n+1} = f(a_n)$). The sum of the first n terms of a sequence can be written using summation notation. Arithmetic sequences have constant differences between terms, which can be represented as a, a+d, a+2d, ... The formula to calculate the common difference is: Common difference = Successive term - Preceding term. In this case, if "a" is the first term and "r" is the common ratio of a geometric sequence, then it is represented by a, ar, ar², ar³, ..., arⁿ⁻¹, ... A sequence is a set of numbers that follow a specific pattern or rule. There are different types of sequences, including arithmetic and geometric sequences. Arithmetic Sequences: An arithmetic sequence is one where each term is obtained by adding a fixed constant to the previous term. The formula for an arithmetic sequence with first term a and common difference d is $\frac{n}{2} \left(2a + (n-1)d \right)$, which can be remembered as "first term plus last term, times the number of terms, divided by two". Geometric Sequences: A geometric sequence is one where each term is obtained by multiplying the previous term by a fixed constant. The formula for a geometric sequence with first term a and common ratio r is $\frac{a(1-r^n)}{1-r}$. Sums of Terms: The sum of the terms in an arithmetic or geometric sequence can be found using various formulas. For example, the sum of the first n terms of an arithmetic sequence with first term a and common difference d is $\frac{n}{2} \left(2a + (n-1)d \right)$. Examples: Some examples of sequences include: * The sequence defined by $a_n = n^2 - n$, where $a_3 = 8$ and $a_{10} = 45$. The difference between consecutive terms is $d = n - 1$. * The sequence defined by $a_n = 0$ and $a_n = \frac{1}{n}$ for $n \geq 1$, where $a_{10} = 298$. * The sequence defined by $a_n = 1$ and $a_n = \frac{1}{n^3}$ for $n \geq 1$, where $a_{10} = 6.666...$ Properties of Sums: The sum of the terms in a sequence can be affected by various properties, such as the common ratio or difference. Convergence: Some sequences converge to a finite value, while others do not. For example, the sequence defined by $a_n = An^2 + Bn + C$ converges if and only if $|r|$