 In Section 14.1, we learned that an exponential function is a function of the form \begin{equation*} f\left(x\right)={b}^{x}\text{,} \end{equation*} where \(b>0\) and \(be 1\text{.}\)We also learned that the graph of an exponential function has the following properties: Table 14.4.1. Properties of the Graph of \(f(x)=b^x\) when \(b>1\) when \(00\text{,}\) \begin{equation*} y=\mathrm{ln}\left(x\right)\text{ is equivalent to }{e}^{y}=x \end{equation*} We read \(\mathrm{ln}\left(x\right)\) as, “the logarithm with base \(e\) of \(x\)” or “the natural logarithm of \(x\text{.}\)” This is called the natural logarithm. Solve \(y=\mathrm{log}\left(10{,}000\right)\) without using a calculator. ExplanationFirst we rewrite the logarithm in exponential form: \(10^y=10{,}000\text{.}\) Next, we ask, “To what exponent must \(10\) be raised in order to get \(10{,}000\text{?}\)” We know \(10^4=10{,}000\text{.}\) Therefore, \(y=\mathrm{log}\left(10{,}000\right)=4\text{.}\) Write the equation \(\mathrm{ln}\left(P\right)=5\) in exponential form. Explanation First, identify the values of \(b\text{,}\) \(y\text{,}\) and \(x\text{.}\) Then, write the equation in the form \(b^y=x\text{.}\) \(\mathrm{ln}\left(P\right)=5\) Here, \(b=e\text{,}\) \(y=5\) and \(x=P\text{.}\) Therefore, the equation \(\mathrm{ln}\left(P\right)=5\) is equivalent to \(e^5=P\text{.}\) In Section 14.3, we learned a formula that would allow us to rewrite logarithmic expressions, which are not of base \(10\) or base \(e\text{,}\) so that we are able to evaluate the expression using a scientific calculator. The Change-of-Base Formula introduces a new base \(a\text{.}\) This can be any base \(a\) we want where \(a\gt 0, aeq 1\text{.}\) Because our calculators have keys for logarithms base \(10\) and base \(e\text{,}\) we will typically choose to write the Change-of-Base Formula with the new base as either \(10\) or \(e\text{.}\) For any logarithmic bases \(a\text{,}\) \(b\text{,}\) \(M\gt 0\) and \(eq 1\text{,}\) Rounding to three decimal places, approximate \(\mathrm{log}_{5}172\text{.}\) Explanation We want to evaluate \(\mathrm{log}_{5}172\text{,}\) which is not base \(10\) or base \(e\text{.}\) So we can't enter it directly into our calculator at this point, but we can use the Change-of-Base Formula to rewrite it into either base \(10\) or base \(e\text{.}\) Then, we will be able to enter it into the calculator. According to the Change-of-Base Formula, \begin{equation*} \mathrm{log}_{b}M=\frac{\mathrm{log}M}{\mathrm{log}b}\text{ or }\mathrm{log}_{b}M=\frac{\mathrm{ln}M}{\mathrm{ln}b} \end{equation*} To use this formula, we need to identify that \(b=5\) and \(M=172\text{.}\) Now let's choose log base \(10\) to rewrite the log. \begin{equation*} \mathrm{log}_{5}172=\frac{\mathrm{log}172}{\mathrm{log}5}\text{.} \end{equation*} Enter the expression \(\frac{\mathrm{log}172}{\mathrm{log}5}\) in the calculator using the log button for base \(10\text{.}\) Then, rounding to three decimal places, we have \begin{equation*} \mathrm{log}_{5}172\approx 3.198\text{.} \end{equation*} For the function \(f(x)= 7^x,\) calculate the following function values: \(f(-3)=\) \(f(-1)=\) \(f(0)=\) \(f(1)=\) \(f(3)=\) For the function \(f(x)= 3^x,\) calculate the following function values: \(f(-3)=\) \(f(-1)=\) \(f(0)=\) \(f(1)=\) \(f(3)=\) In the following exercises, graph each exponential function. \(f(x)=6^x\) \(f(x)=(\frac{1}{2})^x\) Graph the functions on the same coordinate system: \(f\left(x\right)={4}^{x} and g\left(x\right)={4}^{x-1}\text{.}\) The compound interest formula is \(\displaystyle{ A(t) = P(1+\frac{r}{n})^{nt} }\) where \(A(t)\) models the amount of money, \(t\) is the number of years, \(P\) is the initial amount, \(r\) is the annual interest and \(n\) is the number of compoundings per year. Jenny saved \({\$47{,}000.00}\) in an account with an annual interest of \(3.8\%\text{.}\) Answer the following questions: 1) The amount of money in the account would be after \(11\) years, if the interest is compounded yearly. 2) The amount of money in the account would be after \(11\) years, if the interest is compounded quarterly. 3) The amount of money in the account would be after \(11\) years, if the interest is compounded monthly. 4) The amount of money in the account would be after \(11\) years, if the interest is compounded daily (assuming that year has 365 days). The compound interest formula is \(\displaystyle{ A(t) = P(1+\frac{r}{n})^{nt} }\) where \(A(t)\) models the amount of money, \(t\) is the number of years, \(P\) is the initial amount, \(r\) is the annual interest and \(n\) is the number of compoundings per year. Maria saved \({\$16{,}000.00}\) in an account with an annual interest of \(2\%\text{.}\) Answer the following questions: 1) The amount of money in the account would be after \(7\) years, if the interest is compounded yearly. 2) The amount of money in the account would be after \(7\) years, if the interest is compounded quarterly. 3) The amount of money in the account would be after \(7\) years, if the interest is compounded monthly. 4) The amount of money in the account would be after \(7\) years, if the interest is compounded daily (assuming that year has 365 days). In the last ten years, the population of Axtonia has grown at a rate of \(2.3\%\) per year to \(57{,}768\text{.}\) If this rate continues, what will be the population in \(10\) more years? The population will be . In the last twenty years, the population of Charizzo has grown at a rate of \(0.4\%\) per year to \(673{,}426\text{.}\) If this rate continues, what will be the population in \(20\) more years? The population will be . For the following exercises, rewrite each equation in exponential form. \({\mathrm{log}}_{N}\left(75\right)=3\) \({\mathrm{log}}_{9}\left(P\right)=Q\) \(\mathrm{ln}\left(a\right)=b\) \(\mathrm{log}\left(200\right)=H\) For the following exercises, rewrite each equation in logarithmic form. \({10}^{R}=M\) \({6}^{b}=c\) \({X}^{Y}=Z\) \({e}^{\frac{1}{2}}=\sqrt{e}\) For the following exercises, evaluate the logarithmic expression without using a calculator. \({\mathrm{log}}_{27}\left(\sqrt[5]{27}\right)=\) \({\mathrm{log}}_{3}\left(\frac{1}{9}\right)=\) \({\mathrm{log}}_{8}\left(2\right)=\) \({\mathrm{log}}_{4}\left(\frac{1}{64}\right)=\) For the following exercises, evaluate each expression using a calculator. Round to the nearest thousandth. \(\mathrm{ln}\left(15{,}425\right) \approx\) \(\mathrm{log}\left(0.564\right) \approx\) In the following exercises, use the Change-of-Base Formula to approximate each logarithm, rounding to three decimal places. \(\mathrm{log}_{4}914 \approx\) \(\mathrm{log}_{6}470 \approx\) \(\mathrm{log}_{5}0.25 \approx\) \(\mathrm{log}_{15}0.24 \approx\) Mapping based Best Questions (Indian Geography)Starting Soon Mathematical function, denoted exp(x) or e^x This article is about the function f(x) = ex and its generalizations. For functions of the form f(x) = xr, see Power function. For the bivariate function f(x,y) = xy, see Exponentiation. For the representation of scientific numbers, see E notation. ExponentialGraph of the exponential functionGeneral informationGeneral definition exp z = e z {\displaystyle \exp z=e^{z}} Domain, codomain and imageDomain C {\displaystyle \mathbb {C} } Image { (0 , ∞) for z ∈ R C ∖ { 0 } for z ∈ C {\displaystyle {\begin{cases}(0,\infty)&{\text{for }}z\in \mathbb {R} \\\mathbb {C} \setminus \{0\}&{\text{for }}z\in \mathbb {C} \end{cases}}} Specific valuesAt zero1Value at 1eSpecific featuresFixed point−Wn(−1) for n ∈ Z {\displaystyle n\in \mathbb {Z} } Related functionsReciprocal exp (− z) {\displaystyle \exp(-z)} InverseNatural logarithm, Complex logarithmDerivative exp ′ z = exp z {\displaystyle \exp '\!z=\exp z} Antiderivative ∫ exp z d z = exp z + C {\displaystyle \int \exp z\,dz=\exp z+C} Series definitionTaylor series exp z = ∑ n = 0 ∞ z n n ! {\displaystyle \exp z=\sum _{n=0}^{\infty }{\frac {z^{n}}{n!}}} In mathematics, the exponential function is the unique real function which maps zero to one and has a derivative everywhere equal to its value. The exponential of a variable x {\displaystyle x} is denoted exp x {\displaystyle \exp x} or e x {\displaystyle e^{x}} , with the two notations used interchangeably. It is called exponential because its argument can be seen as an exponent to which a constant number e ≈ 2.718, the base, is raised. There are several other definitions of the exponential function, which are all equivalent although being of very different nature. The exponential function converts sums to products: it maps the additive identity 0 to the multiplicative identity 1, and the exponential of a sum is equal to the product of separate exponentials, exp (x + y) = exp x · exp y {\displaystyle \exp(x+y)=\exp x\cdot \exp y} . Its inverse function, the natural logarithm, ln {\displaystyle \ln } or log {\displaystyle \log } , converts products to sums: ln (x · y) = ln x + ln y {\displaystyle \ln(x\cdot y)=\ln x+\ln y} . The exponential function is occasionally called the natural exponential function, matching the name natural logarithm, for distinguishing it from some other functions that are also commonly called exponential functions. These functions include the functions of the form f (x) = b x {\displaystyle f(x)=b^{x}} , which is exponentiation with a fixed base b {\displaystyle b} . More generally, and especially in applications, functions of the general form f (x) = a b x {\displaystyle f(x)=ab^{x}} are also called exponential functions. They grow or decay exponentially in that the rate that f (x) {\displaystyle f(x)} changes when x {\displaystyle x} is increased is proportional to the current value of f (x) {\displaystyle f(x)} . The exponential function can be generalized to accept complex numbers as arguments. This reveals relations between multiplication of complex numbers, rotations in the complex plane, and trigonometry. Euler's formula exp i θ = cos θ + i sin θ {\displaystyle \exp i\theta =\cos \theta +i\sin \theta } expresses and summarizes these relations. The exponential function can be even further generalized to accept other types of arguments, such as matrices and elements of Lie algebras. The graph of y = e x {\displaystyle y=e^{x}} is upward-sloping, and increases faster than every power of x {\displaystyle x} .[1] The graph always lies above the x-axis, but becomes arbitrarily close to it for large negative x; thus, the x-axis is a horizontal asymptote. The equation d d x e x = e x {\displaystyle {\tfrac {d}{dx}}e^{x}=e^{x}} means that the slope of the tangent to the graph at each point is equal to its height (its y-coordinate) at that point. See also: Characterizations of the exponential function There are several equivalent definitions of the exponential function, although of very different nature. The derivative of the exponential function is equal to the value of the function. Since the derivative is the slope of the tangent, this implies that all green right triangles have a base length of 1. One of the simplest definitions is: The exponential function is the unique differentiable function that equals its derivative, and takes the value 1 for the value 0 of its variable. This "conceptual" definition requires a uniqueness proof and an existence proof, but it allows an easy derivation of the main properties of the exponential function. Uniqueness: If f (x) {\displaystyle f(x)} and g (x) {\displaystyle g(x)} are two functions satisfying the above definition, then the derivative of f / g {\displaystyle f/g} is zero everywhere because of the quotient rule. It follows that f / g {\displaystyle f/g} is constant; this constant is 1 since f (0) = g (0) = 1 {\displaystyle f(0)=g(0)=1} . Existence is proved in each of the two following sections. The exponential function is the inverse function of the natural logarithm. The inverse function theorem implies that the natural logarithm has an inverse function, that satisfies the above definition. This is a first proof of existence. Therefore, one has ln (exp x) = x exp (ln y) = y {\displaystyle {\begin{aligned}\ln(\exp x)&=x\\\exp(\ln y)&=y\end{aligned}}} for every real number x {\displaystyle x} and every positive real number y . {\displaystyle y.} The exponential function is the sum of the power series[2][3] exp (x) = 1 + x + x 2 2 ! + x 3 3 ! + ⋯ = ∑ n = 0 ∞ x n n ! , {\displaystyle {\begin{aligned}\exp(x)&=1+x+{\frac {x^{2}}{2!}}+{\frac {x^{3}}{3!}}+\cdots \\&=\sum _{n=0}^{\infty }{\frac {x^{n}}{n!}},\end{aligned}}} The exponential function (in blue), and the sum of the first n + 1 terms of its power series (in red) where n ! {\displaystyle n!} is the factorial of n (the product of the n first positive integers). This series is absolutely convergent for every x {\displaystyle x} per the ratio test. So, the derivative of the sum can be computed by term-by-term differentiation, and this shows that the sum of the series satisfies the above definition. This is a second existence proof, and shows, as a byproduct, that the exponential function is defined for every x {\displaystyle x} , and is everywhere the sum of its Maclaurin series. The exponential satisfies the functional equation: exp (x + y) = exp (x) · exp (y) . {\displaystyle \exp(x+y)=\exp(x)\cdot \exp(y).} This results from the uniqueness and the fact that the function f (x) = exp (x + y) / exp (y) {\displaystyle f(x)=\exp(x+y)/\exp(y)} satisfies the above definition. It can be proved that a function that satisfies this functional equation has the form x ↦ exp (c x) {\displaystyle x\mapsto \exp(cx)} if it is either continuous or monotonic. It is thus differentiable, and equals the exponential function if its derivative at 0 is 1. The exponential function is the limit, as the integer n goes to infinity,[4][3] exp (x) = lim n → + ∞ (1 + x n) n . {\displaystyle \exp(x)=\lim _{n\to +\infty }\left(1+{\frac {x}{n}}\right)^{n}.} By continuity of the logarithm, this can be proved by taking logarithms and proving x = lim n → ∞ ln (1 + x n) n = lim n → ∞ n ln (1 + x n) , {\displaystyle x=\lim _{n\to \infty }\ln \left(1+{\frac {x}{n}}\right)^{n}=\lim _{n\to \infty }n\ln \left(1+{\frac {x}{n}}\right),} for example with Taylor's theorem. Reciprocal: The functional equation implies e x e − x = 1 {\displaystyle e^{x}e^{-x}=1} . Therefore e x ≠ 0 {\displaystyle e^{x}eq 0} for every x {\displaystyle x} and 1 e x = e − x . {\displaystyle {\frac {1}{e^{x}}}=e^{-x}.} Positiveness: e x > 0 {\displaystyle e^{x}>0} for every real number x {\displaystyle x} . This results from the intermediate value theorem, since e 0 = 1 {\displaystyle e^{0}=1} and, if one would have e x < 0 {\displaystyle e^{x}